# Collider Physics 

## Lisboa 2012

I. Basic facts
2. Standard Model examples

## Disclaimer

- The goal is to treat some of the basic elements aiming to make it easier to go to the literature
- Many details will not be mentioned
- Basic reference:Tao Han hep-ph/0508097
- Another reference: "QCD and Collider Physics" by Ellis, Stirling and Webber.


## Motivation

© What we know:

$$
\mathcal{L}=\mathcal{L}_{\text {kinetic }}^{\mathrm{f}}+\mathcal{L}_{\text {kinetic }}^{\mathrm{GB}}+\mathcal{L}_{\mathrm{ffv}}+\mathcal{L}_{\mathrm{vvv}}+\mathcal{L}_{\mathrm{Vvvv}}+\mathcal{L}_{\mathrm{EWSB}}
$$

 bosons tested at $0.1 \%$ level.
© Some information on the interactions between the gauge bosons
(1) $\mathcal{L}_{\text {EWSB }}$ has not been directly tested: origin of masses, flavor physics, ...


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LHC has already enough data to start testing the SM and going beyond it


## I. Basic Facts

- Collider parameters
- $e^{+} e^{-}$colliders
- Hadron colliders
- Detectors
- Useful kinematical variables
- Evaluation of scattering amplitudes


## I. Collider parameters

- Relativity together with quantum mechanics lead to
$\Delta p \Delta t>\frac{\hbar}{c} \square$ only asymptotic states are observable

colliders are essential


## - Basic parameters

I. Center-of-mass energy $\quad 1+2 \Longrightarrow \mathrm{X}$
$\mathbf{s} \equiv \mathbf{E}_{\mathbf{C M}}^{\mathbf{2}} \equiv\left(\mathbf{p}_{\mathbf{1}}+\mathbf{p}_{\mathbf{2}}\right)^{\mathbf{2}}=\left\{\begin{array}{l}\left(E_{1}+E_{2}\right)^{2} \quad \text { in the c.m. frame } \vec{p}_{1}+\vec{p}_{2}=0, \\ m_{1}^{2}+m_{2}^{2}+2\left(E_{1} E_{2}-\vec{p}_{1} \cdot \vec{p}_{2}\right) .\end{array}\right.$

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2. Instantaneous luminosity $\mathcal{L}$ : event rate is proportional to $\sigma$

$$
\mathbf{N}_{\text {events }}=\mathcal{L} \sigma(\mathbf{s})
$$

number of particles

- beams are a collection of bunches

Colliding beam


- Important rule of a thumb: $\sigma \propto \mathbf{1} / \mathbf{E}_{\mathbf{C M}}^{\mathbf{2}} \Longrightarrow \mathcal{L}$ grows as $\simeq \mathbf{E}_{\mathbf{C M}}^{\mathbf{2}}$
- Useful luminosity change of units

$$
10^{33} \mathrm{~cm}^{-2} \mathrm{~s}^{-1}=1 \mathrm{nb}^{-1} \mathrm{~s}^{-1} \approx 10 \mathrm{fb}^{-1} / \text { year }
$$



## II. $\mathrm{e}^{+} \mathrm{e}^{-}$colliders

- Main advantages:
$>\mathrm{e}^{+} \mathrm{e}^{-}$interactions are well understood
$>$ Initial charges are zero $=>$ to produce new states
$>$ Scattering kinematics is well understood/constrained
$>$ In the CM frame all energy available to produce new states
$>$ It is possible to polarize the initial beams.
- Main disadvantages:
$>$ large synchroton radiation => linear machines
$>$ It is easier to produce spin-I states in the s-channel
$>$ There are energy losses by bremstrahlung/beamsstrahlung
> The energy spread needs to be taken into account

$$
\int \mathbf{d} \tau \frac{\mathbf{d} \mathcal{L}}{\mathbf{d} \tau} \sigma(\hat{\mathbf{s}}) \text { with } \tau=\sqrt{\hat{\mathbf{s}}} / \sqrt{\mathbf{s}}
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$$

## - SM processes



## III. Hadron colliders

- protons are much heavier than electrons => higher CM energies
- higher luminosities can be achieved
- protons are composed of quarks and gluons => fewer kinematical constraints
- protons are strongly interacting: collisions are messier
- strong interactions $=>$ large cross sections $\sigma_{\text {total }} \simeq 100 \mathrm{mb}$

III. Hadron collid $\epsilon_{\mathbb{E}^{\mathbf{1 0}^{15}} \quad \text { ppppp crosssections }}$
- protons are much heavier t
- higher luminosities can be a
- protons are composed of c kinematical constraints
- protons are strongly intera
- strong interactions => largi

- QCD factorization theorem: for large transfer momentum we have $\sigma(\mathbf{A B} \rightarrow \mathbf{F} \underset{\sim}{\mathbf{X}})=\sum_{\mathbf{a}, \mathrm{b}} \int_{\text {inclusive }} \mathbf{d \mathbf { x } _ { 1 }} \mathbf{d \mathbf { x } _ { 2 }} \mathbf{f}_{\mathbf{a} / \mathbf{A}}\left(\mathbf{x}_{1}, \mathbf{Q}^{2}\right) \mathbf{f}_{\mathrm{b} / \mathbf{B}}\left(\mathbf{x}_{2}, \mathbf{Q}^{2}\right) \hat{\sigma}(\mathbf{a b} \rightarrow \mathbf{F})$
- $f_{b / B}\left(x, Q^{2}\right)$ is de b parton density in the hadron $B$ carrying $x$ of the momentum
characteristic scale



## Useful quantity:

$$
\begin{aligned}
\sigma(s) & =\sum_{a, b} \int d x_{1} d x_{2} f_{a / A}\left(x_{1}\right) f_{b / B}\left(x_{2}\right) \hat{\sigma}(\hat{s}) \\
& =\sum_{a, b} \int d \tau \int \frac{d x}{x} f_{a / A}(x) f_{b / B}(\tau / x) \hat{\sigma}(\stackrel{\tau}{\tau})
\end{aligned}
$$

where $\tau=\mathbf{x}_{1} \mathbf{x}_{\mathbf{2}}$

$$
\sigma(s)=\sum_{\{i j\}} \int_{\tau_{0}}^{1} \frac{d \tau}{\tau} \cdot \frac{\tau}{\hat{s}} \frac{d \mathcal{L}_{i j}}{d \tau} \cdot[\underbrace{\left.\hat{s} \hat{\sigma}_{i j \rightarrow \alpha}(\hat{s})\right]}_{\text {dimensionless }}
$$

we define the parton-parton luminosity

$$
\frac{\tau}{\hat{s}} \frac{d \mathcal{L}_{i j}}{d \tau} \equiv \frac{\tau / \hat{s}}{1+\delta_{i j}} \int_{\tau}^{1} d x\left[f_{i}^{(a)}(x) f_{j}^{(b)}(\tau / x)+f_{j}^{(a)}(x) f_{i}^{(b)}(\tau / x)\right] / x
$$

CTEQ6L1: gg


## IV. Detectors

- Goal: measure position, time, momentum, energy, type,....
- modern detectors are very complex.

- The signal of a particle depends on its interactions and decay length

$$
\mathbf{d}=(\beta \mathbf{c} \tau) \frac{\mathbf{E}}{\mathbf{M}} \approx(300 \mu \mathbf{m})\left(\frac{\tau}{\mathbf{1 0}^{-12} \mathbf{s}}\right) \frac{\mathbf{E}}{\mathbf{M}}
$$

- There are a few possibilities:
- Fast decay, eg, gluons hadronize in

$$
\mathrm{t}_{\mathrm{h}} \sim 1 / \Lambda_{\mathrm{QCD}} \approx 1 /(200 \mathrm{MeV}) \approx 3.3 \times 10^{-24} \mathrm{~s}
$$

energetic $q / g$ produce jets


- The signal of a particle depends on its interactions and decay length


## ALEPH dali

Run=15768<br>Evt=5906



# - stable particles: $\left(\mathbf{p}, \overline{\mathbf{p}}, \mathbf{e}^{ \pm}, \gamma\right)$ leave energy deposit and/or tracks 

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- Quasi-stable particles: behave like the stable ones

$$
\left(\tau>10^{-\mathbf{1 0}} \text { s, e.g. } \quad \mathbf{n}, \boldsymbol{\Lambda}, \mathbf{K}_{\mathbf{L}}^{\mathbf{0}}, \ldots \mu^{ \pm}, \pi^{ \pm}, \mathbf{K}^{ \pm}, \ldots\right)
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- Short lived resonances decay promptly $\mathbf{W}^{ \pm}, \mathbf{Z}\left(\mathbf{1 0}^{-\mathbf{2 5}} \mathrm{s}\right) ; \pi^{\mathbf{0}}, \rho, \ldots$
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- Displaced vertices:

$$
B^{0, \pm}, D^{0, \pm}, \tau^{ \pm},\left(\tau \sim 10^{-12} \mathrm{~s} ; c \tau \sim 100 \mu \mathrm{~m}\right) . K_{S}^{0} \rightarrow \pi^{+} \pi^{-} \mathrm{w} / c \tau \sim 2.7 \mathrm{~cm}
$$

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$$
\left(\tau>\mathbf{1 0}^{-10} \text { s, e.g. } \mathbf{n}, \boldsymbol{\Lambda}, \mathbf{K}_{\mathbf{L}}^{\mathbf{0}}, \ldots \mu^{ \pm}, \pi^{ \pm}, \mathbf{K}^{ \pm}, \ldots\right)
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$$

- Neutral weakly interacting particles leave no signal $(\nu)$


## - More complex analyses can be made

| Leptons | Vertexing | Tracking | ECAL | HCAL | Muon Cham. |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $e^{ \pm}$ | $\times$ | $\vec{p}$ | $E$ | $\times$ | $\times$ |
| $\mu^{ \pm}$ | $\times$ | $\vec{p}$ | $\sqrt{ }$ | $\sqrt{ }$ | $\vec{p}$ |
| $\tau^{ \pm}$ | $\sqrt{ } \times$ | $\sqrt{ }$ | $e^{ \pm}$ | $h^{ \pm} ; 3 h^{ \pm}$ | $\mu^{ \pm}$ |
| $\nu_{e}, \nu_{\mu}, \nu_{\tau}$ | $\times$ | $\times$ | $\times$ | $\times$ | $\times$ |
| Quarks |  |  |  |  |  |
| $u, d, s$ | $\times$ | $\sqrt{ }$ | $\sqrt{ }$ | $\sqrt{ }$ | $\times$ |
| $c \rightarrow D$ | $\sqrt{ }$ | $\sqrt{ }$ | $e^{ \pm}$ | $h^{\prime}$ s | $\mu^{ \pm}$ |
| $b \rightarrow B$ | $\sqrt{ }$ | $\sqrt{ }$ | $e^{ \pm}$ | $h^{\prime}$ s | $\mu^{ \pm}$ |
| $t \rightarrow b W^{ \pm}$ | $b$ | $\sqrt{ }$ | $e^{ \pm}$ | $b+2$ jets | $\mu^{ \pm}$ |
| Gauge bosons |  |  |  |  |  |
| $\gamma$ | $\times$ | $\times$ | $E$ | $\times$ | $\times$ |
| $g$ | $\times$ | $\sqrt{ }$ | $\sqrt{ }$ | $\sqrt{ }$ | $\times$ |
| $W^{ \pm} \rightarrow \ell^{ \pm} \nu$ | $\times$ | $\vec{p}$ | $e^{ \pm}$ | $\times$ | $\mu^{ \pm}$ |
| $\rightarrow q \bar{q}^{\prime}$ | $\times$ | $\sqrt{ }$ | $\sqrt{ }$ | 2 jets | $\times$ |
| $Z^{0} \rightarrow \ell^{+} \ell^{-}$ | $\times$ | $\vec{p}$ | $e^{ \pm}$ | $\times$ | $\mu^{ \pm}$ |
| $\rightarrow q \bar{q}$ | $(b \bar{b})$ | $\sqrt{ }$ | $\sqrt{ }$ | 2 jets | $\times$ |

- Typical detector performance:
$>$ Coverage: $\quad\left|\eta_{\text {track }}\right|<2.5 \quad\left|\eta_{\text {cal }}\right|<5$.
> Tracker momentum resolution

$$
\frac{\Delta \mathbf{p}_{\mathbf{T}}}{\mathbf{p}_{\mathbf{T}}}=\mathbf{0 . 3 6} \mathbf{p}_{\mathbf{T}} \oplus \frac{\mathbf{0 . 0 1 3}}{\sqrt{\sin \theta}}(\text { in } \mathrm{TeV})
$$

> ECAL resolution:

$$
\frac{\Delta \mathbf{E}}{\mathbf{E}}=\frac{\mathbf{1 0 \%}}{\sqrt{\mathbf{E} / \mathrm{GeV}}} \oplus \mathbf{0 . 4 \%}
$$

> HCAL resolution

$$
\frac{\mathbf{\Delta E}}{\mathbf{E}}=\frac{\mathbf{8 0 \%}}{\sqrt{\mathbf{E} / \mathrm{GeV}}} \oplus \mathbf{1 5} \%
$$

## > Vertexing performance:

$$
\mathbf{A d}_{\mathbf{0}}=11 \oplus \frac{7 \mathbf{3}}{(\mathbf{p} \mathbf{T} / \mathrm{GeV}) \sqrt{\sin \theta}}(\mu \mathrm{m})
$$

$\Delta \mathrm{z}_{\mathbf{0}}=87 \oplus \frac{115}{\left(\mathrm{p}_{\mathbf{T}} / \mathrm{GeV}\right) \sqrt{\sin ^{3} \theta}}(\mu \mathrm{~m})$
> Vertexing performance:

$$
\Delta \mathbf{d}_{\mathbf{0}}=11 \oplus \frac{73}{\left(\mathbf{p}_{\mathbf{T}} / \mathrm{GeV}\right) \sqrt{\sin \theta}}(\mu \mathrm{m})
$$



Primary vertex



Event with 20 reconstructed vertices (ellipses have $20 \sigma$ size for visibility reasons)

- Trigger: for large events rates, eg, at LHC it is 40 MHz , it is impossible to store all events.
- At the LHC a event rate of 200 Hz can be stored!!!
- the trigger is a fast selection to reduce the event rate for writing.
- There are several layers of decision (level-I, level-2, etc)



## Useful kinematical variables

- Subprocess center-of-mass energy: in the LAB frame

$$
\mathbf{p}_{\mathrm{CM}}^{\mu}=\simeq \frac{\sqrt{\mathrm{s}}}{2}\left(\mathbf{x}_{1}+\mathbf{x}_{2}, \mathbf{0}, \mathbf{0}, \mathbf{x}_{1}-\mathbf{x}_{2}\right) \quad \Longrightarrow \hat{\mathbf{s}}=\mathbf{x}_{1} \mathbf{x}_{2} \mathbf{s}
$$

- Rapidity/pseudo-rapidity: $\mathbf{E}(\mathbf{1}, \beta \sin \theta \cos \phi, \beta \sin \theta \sin \phi, \beta \cos \theta)$

$$
\mathbf{y} \equiv \frac{\mathbf{1}}{\mathbf{2}} \log \frac{\mathbf{E}+\mathbf{p}_{\mathbf{z}}}{\mathbf{E}-\mathbf{p}_{\mathbf{z}}} \longrightarrow \eta=\frac{\mathbf{1}}{\mathbf{2}} \log \frac{\mathbf{1}+\cos \theta}{\mathbf{1}-\cos \theta} \text { for } \beta \rightarrow \mathbf{1}
$$

- The CM and LAB frames are related by

$$
\mathbf{y}=\mathbf{y}^{*}+\underset{\text { center-of-mass rapidity }}{\mathbf{y} . \mathbf{m} .}=\mathbf{y}^{*}+\frac{\mathbf{1}}{\mathbf{2}} \log \frac{\mathbf{x}_{\mathbf{1}}}{\mathbf{x}_{\mathbf{2}}}
$$



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$$

- A useful change of variables is
$\mathbf{x}_{1,2}=\sqrt{\tau} \mathbf{e}^{ \pm \mathrm{y}_{\mathrm{cm}}} \Longrightarrow \int_{\tau_{0}}^{1} \mathrm{dx}_{1} \int_{\tau_{0} / \mathrm{x}_{1}}^{1} \mathrm{dx}_{2}=\int_{\tau_{0}}^{1} \mathrm{~d} \tau \int_{\frac{1}{2} \ln \tau}^{-\frac{1}{2} \ln \tau} \mathrm{~d}_{\mathrm{cm}}$
- Largely used due to $\frac{\mathbf{d}^{3} \tilde{\mathbf{p}}}{\mathbf{E}}=\operatorname{dpxdpy} \frac{\mathbf{d p}_{\mathbf{z}}}{\mathbf{E}}=\mathbf{p}_{\mathbf{T}} \mathbf{d p}_{\mathbf{T}} \mathbf{d} \varphi \mathrm{dy}$
with the $\varphi$ (azimuthal angle), $\mathbf{p}_{\mathbf{T}}$ (transverse momentum) and $\mathbf{y}$ being invariant under longitudinal boosts
- It is usual to represent deposit of energy in the $(\eta, \varphi)$ plane
and the separation $\quad \Delta \mathbf{R}=\sqrt{\Delta \varphi^{2}+\Delta \eta^{2}}$


# GAILAS EXPERIMENT 

$$
Z \rightarrow \mu^{-} \mu^{+}+3 \text { jets }
$$

Run Number 158466, Event Number 4174272 Date: 2010-07-02 17:49:13 CEST


## Invariant mass

- Consider an unstable particle $\left(\mathbf{X}=\mathbf{Z}, \mathbf{W}^{ \pm}, \mathbf{t}\right)$ decaying $\mathbf{X}^{\operatorname{san} \boldsymbol{2}} \rightarrow \mathbf{a b} \ldots$

$$
\frac{\mathbf{d} \sigma}{\mathrm{dM}_{\mathrm{ab} \ldots}} \propto \frac{1}{\left(\mathbf{M}_{\mathrm{ab} \ldots}^{2}-\mathbf{M}_{\mathrm{X}}^{2}\right)^{2}+\Gamma_{\mathrm{X}}^{2} \mathbf{M}_{\mathrm{X}}^{2}}
$$

and exhibits a peak for $\mathbf{M}_{\mathbf{a b} \ldots}^{2}=\left(\mathbf{p}_{\mathbf{a}}+\mathbf{p}_{\mathbf{b}}+\ldots\right)^{\mathbf{2}}=\left(\sum_{\mathbf{i}}^{\mathrm{n}} \mathbf{p}_{\mathbf{i}}\right)^{\mathbf{2}} \approx \mathbf{M}_{\mathbf{X}}^{2}$

- For the same reason the production $\mathbf{a b} \rightarrow \mathbf{X}+$ anything exhibits a peak for $\mathbf{M}_{\mathrm{ab}}^{2} \simeq \mathbf{M}_{\mathbf{X}}^{2}$
- If the decays products are observable $\Longrightarrow$ we can reconstruct $\mathbf{M a b}_{\mathbf{a b}}$.., e.g. $\mathbf{Z} \rightarrow \mathbf{e}^{+} \mathbf{e}^{-}, \mathbf{b} \overline{\mathbf{b}}, \ldots$
- $\mathbf{e}^{+} \mathbf{e}^{-} \rightarrow \mathbf{Z}:$ in this case $\mathbf{M}_{\mathbf{e}^{+} \mathbf{e}^{-}}=\sqrt{\mathbf{s}}$

- At the CMS $p p \rightarrow \mathbf{Z}+\mathbf{X} \rightarrow \mu^{+} \mu^{-}$




## Transverse mass

- Consider the process $\mathbf{p} \overline{\mathbf{p}} \rightarrow \mathbf{W X} \rightarrow \mathbf{e} \nu \mathbf{X}$

$$
\mathbf{m}_{\mathbf{e} \nu}^{2}=\left(\mathbf{E}_{\mathbf{e}}+\mathbf{E}_{\nu}\right)^{2}-\left(\tilde{\mathbf{p}}_{\mathbf{e} \mathbf{T}}+\tilde{\mathbf{p}}_{\nu \mathbf{T}}\right)^{2}-\left(\mathbf{p}_{\mathbf{e z}}+\mathbf{p}_{\nu \mathbf{Z}}\right)^{2}
$$

However, $\tilde{\mathbf{p}}_{\nu}$ is not observable.

- We can infer $\vec{p}_{\nu T} \simeq \tilde{p_{T}}=-\sum \tilde{\mathbf{p}_{\mathbf{T}}}$ (observed). Analogously $\ddot{B}_{T}=\mathbf{E}_{\nu}$
uses all we can measure in the transverse plane
- We define the transverse mass [UAI]
$\mathbf{m}_{\mathrm{e} \nu \mathbf{T}}^{2} \equiv\left(\mathbf{E}_{\mathrm{eT}}+\mathbf{E}_{\nu \mathbf{T}}\right)^{2}-\left(\tilde{\mathbf{p}}_{\mathrm{e} \mathbf{T}}+\tilde{\mathbf{p}}_{\nu \mathbf{T}}\right)^{2} \approx 2 \tilde{\mathbf{p}}_{\mathrm{e} \mathbf{T}} \cdot \tilde{\mathbf{p}}_{\nu \mathbf{T}} \approx 2 \mathbf{E}_{\mathbf{e T}} \mathscr{H}_{T}\left(1-\cos \phi_{\mathbf{e} \nu}\right)$
- In general $\mathbf{0} \leq \mathbf{m}_{\mathbf{e} \nu \mathbf{T}} \leq \mathbf{m}_{\mathbf{e} \nu}$ (Prove it!)
- For $q \bar{q}^{\prime} \rightarrow W^{*} \rightarrow e \nu$ there is a Jacobian peak.




W signature:

- Isolated high- $p_{T} e / \mu$
- missing $E_{T}$

theory must match level of precison


## W mass at hadron colliders



W sigr

- Isola 1
- missi


Hadronic recoil

MET


W signature:

- Isolated high- $p_{T} e / \mu$
- missing $E_{T}$



W signature:

- Isolated high- $p_{T} e / \mu$
- missing $E_{T}$


Main backgrounds:

- QCD multijet
$-Z \rightarrow \ell$
- $W \rightarrow \tau \nu \rightarrow \ell \nu \nu \nu$

Kinematical variables:

- transverse mass

$$
m_{T}=\sqrt{2 E_{T}^{\ell} E_{T}\left(1-\cos \varphi_{\ell \nu}\right)}
$$

- lepton transverse momentum $E_{T}$

Kinematical variables:

- trans
$m_{T}$
- leptc
$E_{T}$


漛 Rather insensitive to $\overrightarrow{\mathbf{p}}_{\mathrm{W}}$畨 $\mathrm{m}_{\mathrm{T}}$ has a significant resolution sensitivity

Kinematical variables:

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$$
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## Kinematical variables:

- transverse mass
$m_{T}$

$$
\frac{\mathrm{d} \hat{\sigma}}{\mathrm{~d} \mathrm{p}_{\mathrm{eT}}}=\frac{4 \mathrm{p}_{\mathrm{eT}}}{\hat{\mathbf{s}} \sqrt{1-4 \mathrm{p}_{\mathrm{eT}}^{2} / \hat{\mathrm{s}}}} \frac{\mathrm{~d} \hat{\sigma}}{\mathrm{~d} \cos \theta^{*}}
$$

there is a Jacobian peak at $\mathbf{p}_{\mathbf{e T}}=\mathbf{M}_{\mathbf{W}} / \mathbf{2}$

** small detector smearing effect
畨 significant $p_{T}^{W}$ effect

Kinematical variables:

- transverse mass

$$
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## D0 event selection:

- $P_{T}^{e}>25 \mathrm{GeV}$
- $\left|\eta_{e}\right|<1.05$
- $\mathscr{E}_{T}>25 \mathrm{GeV}$
$-50<m_{T}<200 \mathrm{GeV}$
$-E_{T}^{\text {had }}<15 \mathrm{GeV}$


## $M_{W}$ and $\Gamma_{W}$ are measured fitting the distributions

## $\mathrm{m}_{\mathrm{T}}$ method


$\mathrm{m}_{\mathrm{W}}=80.401 \pm 0.023 \mathrm{GeV}$ (stat)
Fit range: $65<m_{T}<90 \mathrm{GeV}$

## Electron $\mathrm{p}_{\mathrm{T}}$ method


$\mathrm{m}_{\mathrm{W}}=80.400 \pm 0.027 \mathrm{GeV}$ (stat)
Fit range: $32<p_{T}^{e}<48 \mathrm{GeV}$

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## $M_{W}$ and $\Gamma_{W}$ are measured fitting the distributions



## Motivation:

"Elements" of a collision


## VI. Cross section evaluation

業 Evaluating cross section in a hadron-hadron machine requires
$\sigma=\int d x_{1} d x_{2} \sum_{\text {subp }} f_{a_{1} / p}\left(x_{1}\right) f_{a_{2} / \bar{p}}\left(x_{2}\right)$
$\frac{1}{2 \hat{s}(2 \pi)^{3 n-4}} \int d \Phi_{n}\left(x_{1} P_{A}+x_{2} P_{B} ; p_{1} \ldots p_{n}\right) \Theta($ cuts $) \bar{\sum}|\mathcal{M}|^{2}\left(a_{1} a_{2} \rightarrow b_{1} \ldots b_{n}\right)$

scattering amplitude
we need to evaluate as precisely as we can the cross section

## Phase space

業 The sum of final states is

$$
d \Phi_{n}(a b \rightarrow 1 \ldots n) \equiv \delta^{4}\left(p_{a}+p_{b}-p_{1}-\ldots-p_{n}\right) \prod_{i=1}^{n} \frac{d^{3} \vec{p}_{i}}{2 E_{i}}
$$

Lorentz invariant

蒌 $3 n-4$ integrals．
業 With azimuthal symmetry
$\Longrightarrow 3 n-5$ integrals
潾 In a hadron collider we have 2 extra integrals $\left(x_{1,2}\right)$

| $n$ | $3 n-3$ |
| :---: | :---: |
| 2 | 3 |
| 3 | 6 |
| 4 | 9 |
| $\ldots$ | $\ldots$ |
| 8 | 21 |

## Scattering amplitude evaluation

＊We also need to evaluate $\bar{\Sigma}|\mathcal{M}|^{2}\left(a_{1} a_{2} \rightarrow b_{1} \ldots b_{n}\right)$ with $\mathcal{M}=\sum_{i=1}^{f} \mathcal{M}_{i}$ ．
＊＊If $f(n)$ is large the＂trace technique＂becomes useless since we have to evaluate $f(f+1) / 2$ cross terms $\operatorname{Re}\left(\mathcal{M}_{i}^{*} \mathcal{M}_{j}\right)$ ．

洮 It then becomes advantageous to numerically evaluate $\mathcal{M}_{i} \Longrightarrow$ complexity grows linearly with $f$ ．

湶 One efficient technique is to work in helicity basis

$$
|\mathcal{M}|^{2}=\sum_{\lambda_{a} \ldots \lambda_{n}}\left|\mathcal{M}\left(\lambda_{a} \ldots \lambda_{n}\right)\right|^{2}
$$

摂 For fermions
in the representation $\gamma_{5}=\left(\begin{array}{rr}-1 & 0 \\ 0 & 1\end{array}\right)$ we write $\psi=\binom{\psi_{-}}{\psi_{+}}$
where $\psi_{-}$and $\psi_{+}$are Weyl spinors of negative and positive helicity．
＊＊For instance，$u$－spinor with chiral components $u(p, \sigma)_{ \pm}=\sqrt{p^{0} \pm \sigma|\mathbf{p}|} \chi_{\sigma}(p)$ ， where

$$
\chi_{+}(p)=\frac{1}{\sqrt{2|\mathbf{p}|\left(|\mathbf{p}|+p_{z}\right)}}\binom{|\mathbf{p}|+p_{z}}{p_{x}+i p_{y}} ; \chi_{-}(p)=\frac{1}{\sqrt{2|\mathbf{p}|\left(|\mathbf{p}|+p_{z}\right)}}\binom{-p_{x}+i p_{y}}{|\mathbf{p}|+p_{z}}
$$

脊 The HELAS package has all elements need to evaluate Feynman diagrams defined as fortran routines．For instance，an incoming $u(p, N H)$－spinor is given by a simple subroutine call，

> call IXXXXX(P,FMASS,NH,+1,PSI)
to compute the spinor $v$ change $+1 \rightarrow-1$ ．
摂 Outgoing spinors are generate by call IXXXXX（P，FMASS，NH，$\pm 1$, PSI）
来 the polarization vector of incoming vector bosons is
call VXXXXXXX（P，VMASS，NHEL，-1, VC）

* The package MADGRAPH can be used to generate SM and SUSY amplitudes!
- MADGRAPH can generate $2 \rightarrow 8$ processes
- MADGRAPH already sums over polarizations and colors
- MADGRAPH produces a ps file with the Feynman diagrams
- The package MADEVENT goes further and produces a complete Monte Carlo
- Interfaces for PYTHIA, HERWIG, and ROOT are available

> http://madgraph.hep.uiuc.edu/

